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NONLINEAR OPTIMIZATION AND GENERALIZED LAGRANGE MULTIPLIERS.(U)
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"Nonlinear Optimization and Generalized Lagrange Multipliers"

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Department of Mathematics
University of Washingtion, Seattle
February 19, 1977

A total of 31 research articles and one monograph were published under this grant. These can be grouped in the following categories: nonlinear programming (9 articles), stochastic programming (9 articles), optimal control (8 articles) and supporting areas of convex analysis (5 articles and monograph). Most of these have already been described in the yearly Interim Scientific Reports, so to avoid unnecessary duplication this report will be in the nature of a summary.

Nonlinear Programming [2], [9], [10], [11], [12], [19], [25], [26], [30]. In 1968, Hestenes and Powell independently suggested computational approach to nonlinear programming with equality constraints in which the ordinary Lagrangian function was augmented by quadratic penalty-like terms. Augmented Lagrangians were generalized to inequality-constrained problems in [2], [9], and theory of how they allow saddlepoint characterizations of optimality even in nonconvex programming was presented in [11], and [19]. These papers also explored a new class of algorithms containing the one of Hestenes and Powell as well as the usual quadratic exterior penalty methods as special cases. These algorithms, called "multipliers methods", were explored further in [10], [12], [25] and [30]. They convert a constrained problem into a sequence of unconstrained problems, but without the drawbacks of numerical instability that are associated with classical penalty approaches. They are now regarded among the best methods for nonlinear programming and are the focus of much research by many people.

Paper [26] is an exposition of Lagrange multiplier theory designed to popularize augmented Lagrangians and stimulate further work on saddlepoint characterizations of optimality.

Stochastic Programming [14], [15], [17], [18], [20], [21], [24], [27], [29]. Most of this work was done jointly with R. J. B. Wets, formerly of Boeing Scientific Research Laboratories and now at the University of Kentucky. The objective was to develop (for the first time) a general theory of Lagrange multipliers and necessary and sufficient optimality conditions for optimization problems involving an alternating sequence of decisions and observations of random variables. Judging from experience in other areas, such a theory should be crucial to progress in computation, and the convex case ought to play a central role. The first version of the theory used "measures" as Lagrange multipliers. This was explored in [14], [18], [20], [29]. The case of two-stage convex problems, which has many applications of interest and covers most of the models previously studied in the literature, such as stochastic linear programming, was then subjected to a concentrated attack in [15], [17], [21], [24]. It was found that the "singular multipliers" present in general formulations of the theory could be avoided by a natural and appealing assumption of "relatively complete recourse". This idea was extended to N-stage problems with abstract constraints in [27]. It will be studied in the context of more specialized models in the future.

Optimal Control [4], [5], [6], [7], [16], [22], [23], [31]. Further progress on the theory of duality in control problems with convex costs and constraints was made in [4] (partly summarized in [7]) and [31]. This includes the first complete description of necessary and sufficient conditions for optimality in such problems when state constraints may be active. The results were extended in [6] and [22] to "infinite horizon" problems of a sort common in economic models and certain engineering applications. Very powerful existence theorems for a broad class of control problems were formulated and proved in [5] and [16]. A "semigroup" version of these results on duality and existence was investigated in [23].

Convex Analysis and Integral Functionals [1], [3], [8], [13], [28], [32]. Optimal control problems and stochastic programming problems involve costs and constraints expressed by integrating over time or taking expectations with respect to probability measures. Their analysis therefore depends heavily on properties of "integral functionals" defined on various function spaces, especially kinds of continuity and compactness, which in turn entail convexity, duality and extensions of the theory of measurability (to ensure that functionals are mathematically well-defined, minima attained, etc.). Some of the needed properties were developed in [1] and [3]. More recently, a long and comprehensive article [32] was put together on this subject, with many new results in a framework highly adaptible to applications to diverse problems of optimization. It will be used heavily in future work.

Paper [8] treats a class of dynamic programming problems connected with economic models of production. A growth property useful in the study of Hamiltonian dynamical systems (which occur in the convex control theory referred to above) is described in [28].

The monograph [13] presents the general theory of duality and generalized Lagrange multipliers in optimization problems of convex type. It supplements the writer's earlier book Convex Analysis by covering the infinite-dimensional case and placing more emphasis on saddlepoint optimality.

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19. "Solving a nonlinear programming problem by way of a dual problem", Symposia Mathematica, to appear.

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